

Weekly Focus: solids
Weekly Skill: volume, surface area

LESSON 48: Pyramids, Cones, and Spheres

Lesson Summary: For the warm up, students will solve a problem about Lake Superior. In Activity 1, they will calculate the volume and surface area of spheres. In Activity 2, students will calculate the volume of pyramids and cones. In Activity 3, students they solve for the surface area of pyramids and cones. In Activity 4, they will do word problems. Activity 5 is an application problem about a big sinkhole that occurred in Guatemala. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 48:

- The geometry notes come from: <http://www.asu.edu/courses/mat142ej/geometry/Geometry.pdf> (pages 22 – 26)
- Video A (length 5:15) on volume of pyramids, cones, and spheres
Video B (length 8:30) on surface area of pyramids, cones, and spheres
The videos are required for teachers and optional for students
- 3 Worksheets (48.1, 48.2, 48.3) with answers (attached)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Student Book* (pages 108 – 109)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook* (pages 154 – 157)
- The application activity comes from the site (link embedded in lesson plan)

Objectives: Students will be able to:

- Solve the word problem about Lake Superior
- Calculate the volume and surface area of spheres, cones, and pyramids
- Solve word problems about these solids
- Solve a real-life problem about a sinkhole that happened in Guatemala

ACES Skills Addressed: N, CT, LS, EC

CCRS Mathematical Practices Addressed: Model with Math, Mathematical Fluency, Use Tools Strategically

Levels of Knowing Math Addressed: Intuitive, Pictorial, Abstract, and Application

Notes:

You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The “easier” problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

Lesson 48: Pyramids, Cones, and Spheres

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 48 Warm-up: Solve the Lake Superior Questions

Time: 5-10 Minutes

Write on the board: Lake Superior is located between Canada and the U.S. There are about 175,000 Canadians living along the lake's northern border and about 425,000 Americans along its southern border.

Basic Questions:

- What is the ratio of Canadians to Americans living along Lake Superior?
 - $\frac{175,000}{425,000} = \frac{7}{17}$
- Americans consist of what percent of those living along Lake Superior?
 - $\frac{425,000}{600,000} = \frac{425}{600} = 71\%$

Extension Question:

- The volume of water in Lake Superior is 3.0×10^{15} gallons. Write this number in standard form.
 - 3,000,000,000,000,000 (3 quadrillion gallons)
 - Note: This enough water to cover all of South and North America in one foot of water!

Lesson 48 Activity 1: Volume and Surface Area of a Sphere

Time: 15 Minutes

1. Use the attached **Notes, pages 22 – 26**, for reference.
2. A sphere is a round figure with no bases or faces; a ball is a good example.
3. The formulas for volume and surface area of a sphere are similar to each other. See below.
4. Solve the volume and SA of the spheres on **Worksheet 48.1**. Do #1 and #9 as examples. If the diameter is given, it must be divided by two to get the radius.
5. Students can do some of the problems in class and complete the rest for homework.

Surface area and volume of a:

| | | |
|-------------------------|---------------------------|----------------------------|
| rectangular/right prism | $SA = ph + 2B$ | $V = Bh$ |
| cylinder | $SA = 2\pi rh + 2\pi r^2$ | $V = \pi r^2 h$ |
| pyramid | $SA = \frac{1}{2}ps + B$ | $V = \frac{1}{3}Bh$ |
| cone | $SA = \pi rs + \pi r^2$ | $V = \frac{1}{3}\pi r^2 h$ |
| sphere | $SA = 4\pi r^2$ | $V = \frac{4}{3}\pi r^3$ |

(p = perimeter of base with area B ; $\pi \approx 3.14$)

Lesson 48: Pyramids, Cones, and Spheres

Lesson 48 Activity 2: Volume of Pyramids and Cones

Time: 20 Minutes

1. The objective of this activity is to find the volume of cones and pyramids.
2. These two are grouped together because their formulas are similar. For both, the volume = $\frac{1}{3}$ area of base x height.
3. A **cone** is a solid 3-dimensional object with a circular base and one vertex (point). A good example is an ice cream cone (without the ice cream in it).
4. Since a cone has a circle as the base, we use $\frac{1}{3}$ times πr^2 for the area of the base and multiply it by the height.
5. A **pyramid** is a solid 3-dimensional object with a polygon base and triangular sides that meet at an apex (point). A pyramid can be a square pyramid, a triangular pyramid, a pentagonal pyramid, etc.
6. The formula for the volume of a pyramid is $\frac{1}{3} B$, the area of its base, multiplied by its height.
7. Do #1 and #2 on **Worksheet 48.2** as examples. Then students can do the rest on their own.
8. Volunteers can do a few problems on the board.

Lesson 48 Activity 3: Surface Area of Pyramids and Cones

Time: 20 Minutes

1. The objective of this activity is to calculate the SA of pyramids and cones.
2. The **lateral area**, as referred to in videos and worksheets, means the sum of the area of all surfaces except for the top and bottom bases.
3. The **slant height**, referred to as **s** or **l** on videos or worksheets, is the height of the face, or the height of the triangles on the pyramid. The **s** is used on the GED formula sheet.
4. Finding the surface area of the pyramid means finding the area of each triangle and adding the area of the base.
5. Use #10 on **Worksheet 48.3** as an example. Have the students find the area of one triangle, multiply it by 4, and add the area of the base. **They should get $\frac{1}{2} (4)(6.3)(4) + 16 = 66.4$.**
6. The formula given on the GED test gets the same results and is a little shorter, but it is important for students to understand what they are doing when they calculate the surface area.
7. The SA of a cone is $\pi r s + \pi r^2 = \pi$ multiplied by the radius by the slant height of the cone + the area of the circular base.
8. Use #1 on **Worksheet 48.3** as an example. The radius is 13 and the slant height is 30. Using the formula, we get **$(3.14)(13)(30) + (3.14)(13^2) = 1224.6 + 530.66 = 1755.26 \text{ units}^2$.**
9. Questions #1-6 on the worksheet are meant to label the parts of the figures correctly, and questions #7-12 are to solve. You can have students solve all of them for SA if you have time.
10. Have volunteers do 1-2 problems on the board.

Lesson 48: Pyramids, Cones, and Spheres

Lesson 48 Activity 4: Word Problems

Time: 45 Minutes

1. Do the problems on **pages 108-109** of the **student book** together.
2. Have students do the problems in the **workbook pages 154-157** independently.
3. Volunteers can solve a few of the more challenging problems on the board.

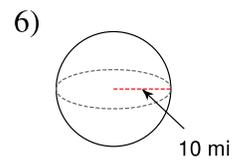
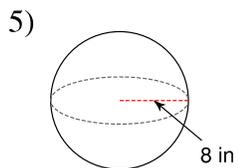
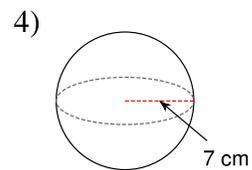
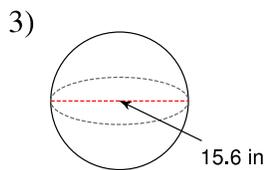
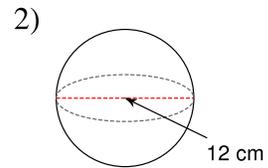
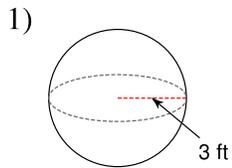
Lesson 48 Application: Guatemalan Sinkhole

Time: 15 Minutes

1. Become familiar with [the activity](#) before class.
2. Show the students the photo of the sinkhole, which will foster good discussion.
3. Follow the suggested format of asking questions and discussion as time permits.
4. The purpose of an activity such as this one is for students to see there are real life applications of the formulas and to make it interesting.

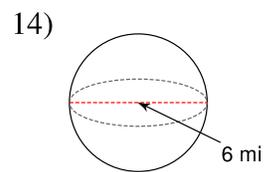
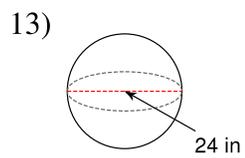
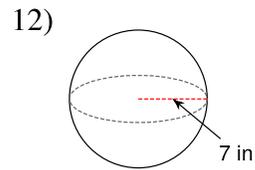
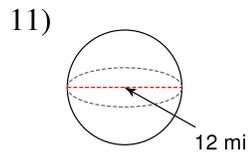
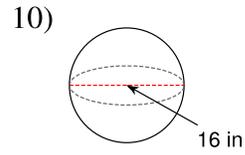
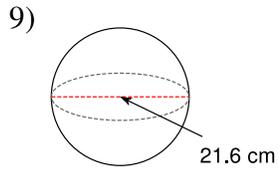
Worksheet 48.1 Spheres

Find the surface area of each figure. Round your answers to the nearest tenth, if necessary.



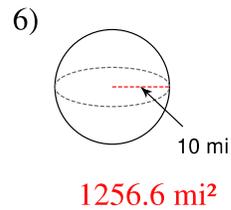
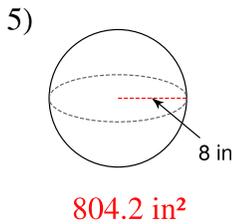
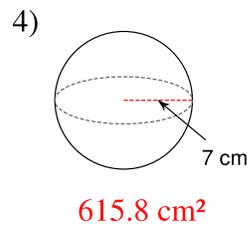
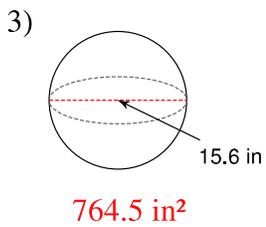
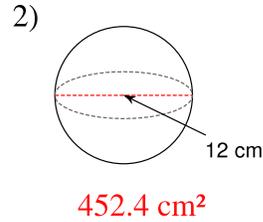
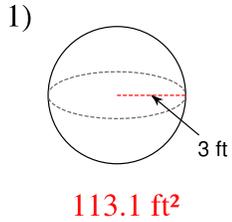
Lesson 48: Pyramids, Cones, and Spheres

Find the volume of each figure. Round your answers to the nearest tenth, if necessary.



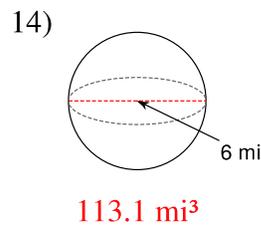
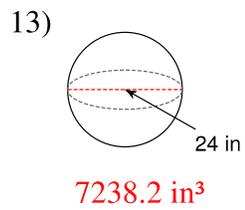
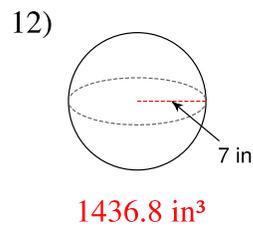
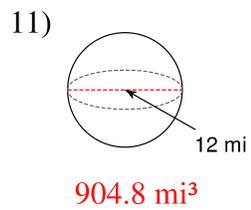
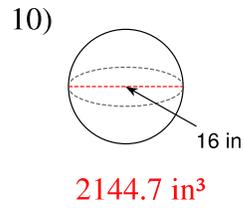
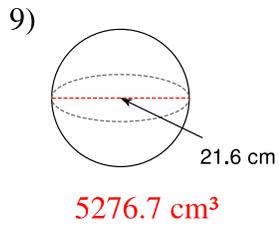
Worksheet 48.1 Answers

Find the surface area of each figure. Round your answers to the nearest tenth, if necessary.



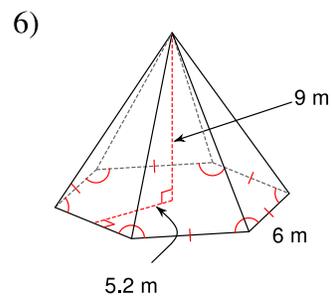
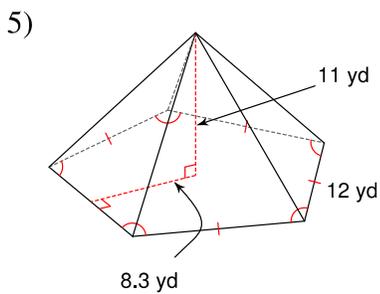
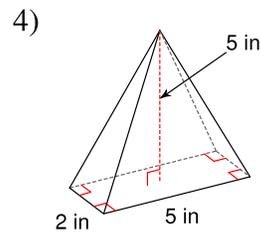
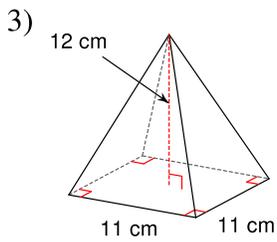
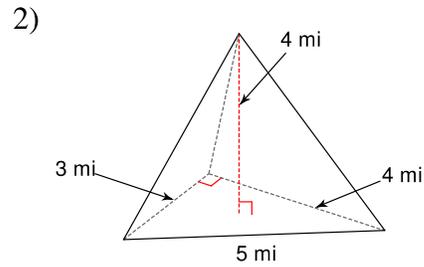
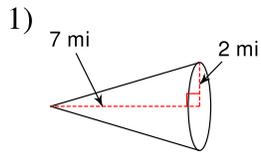
Lesson 48: Pyramids, Cones, and Spheres

Find the volume of each figure. Round your answers to the nearest tenth, if necessary.



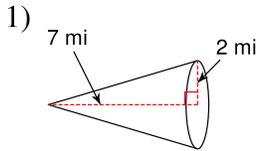
Worksheet 48.2 Volume of Pyramids and Cones

Find the volume of each figure. Round your answers to the nearest tenth, if necessary.

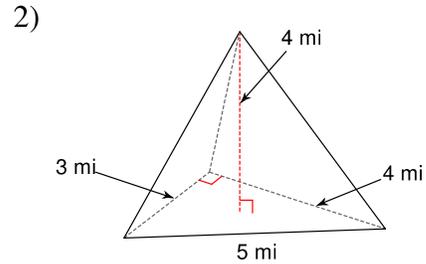


Worksheet 48.2 Answers

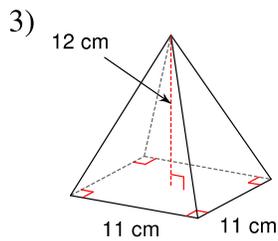
Find the volume of each figure. Round your answers to the nearest tenth, if necessary.



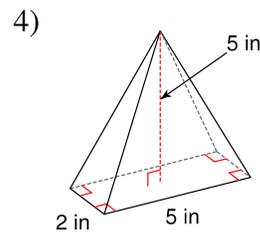
29.3 mi³



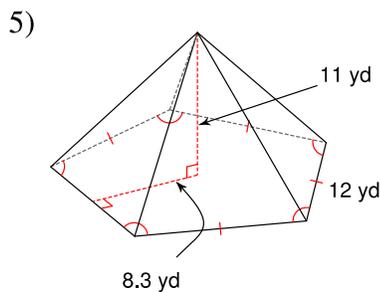
8 mi³



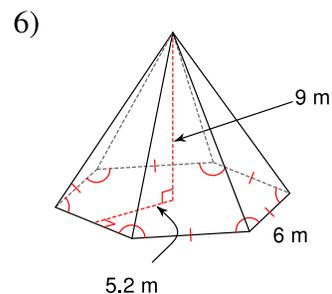
484 cm³



16.7 in³



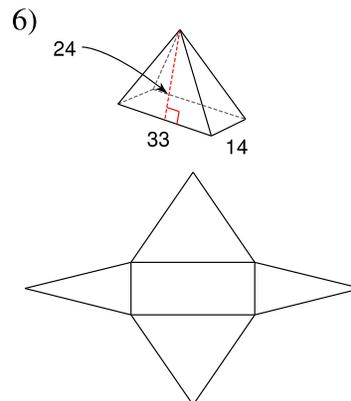
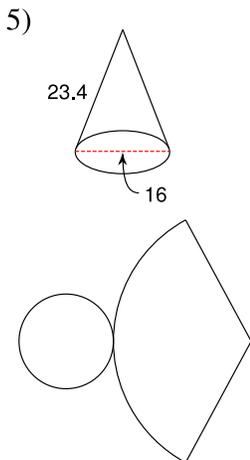
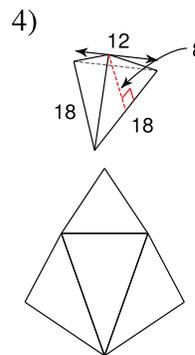
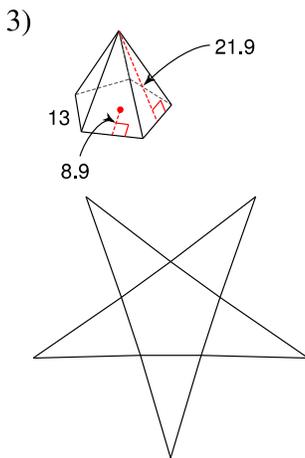
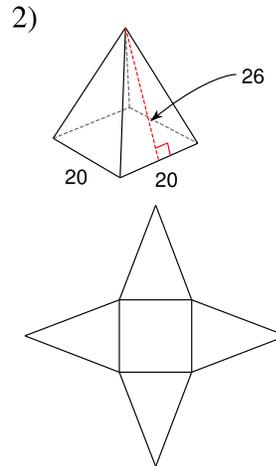
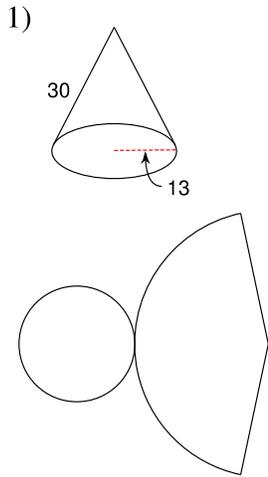
913 yd³



280.8 m³

Worksheet 48.3 Surface Area of Pyramids and Cones

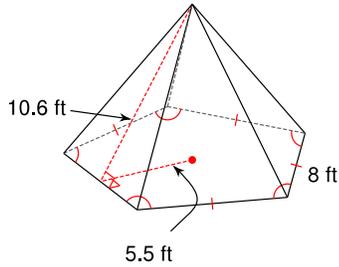
Copy the measurements given onto the net of each solid.



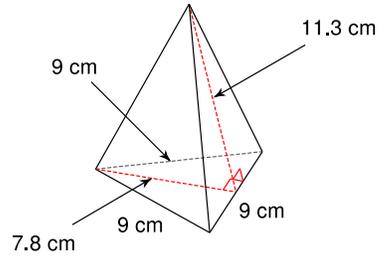
Lesson 48: Pyramids, Cones, and Spheres

Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

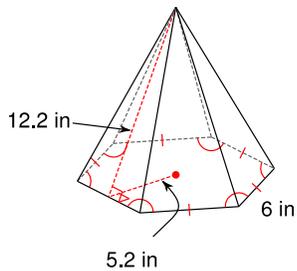
7)



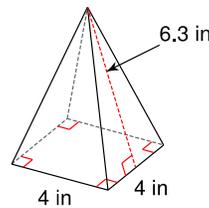
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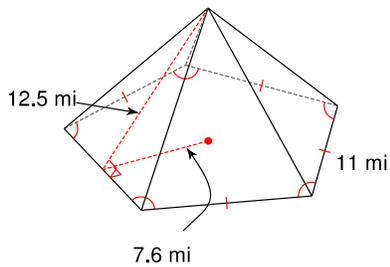
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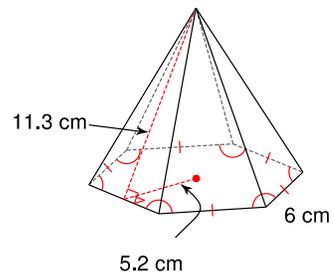
10)



11)

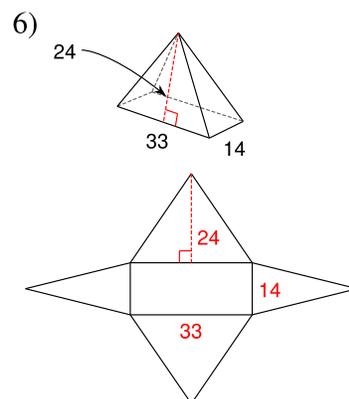
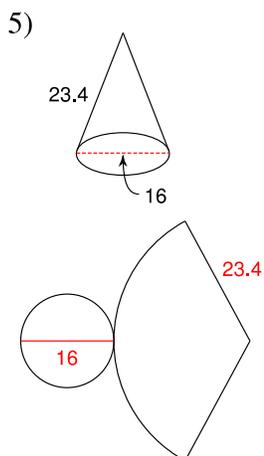
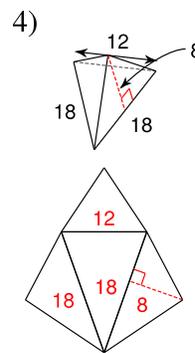
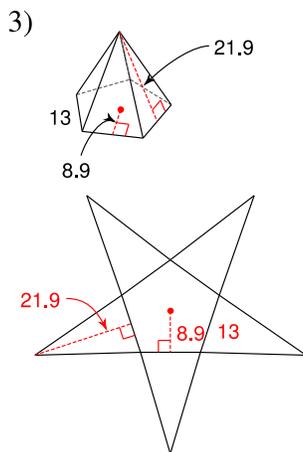
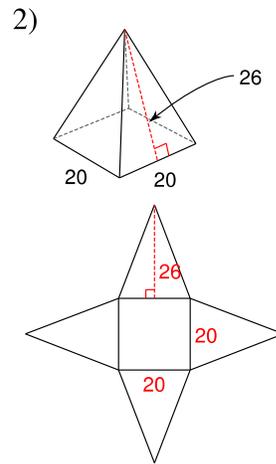
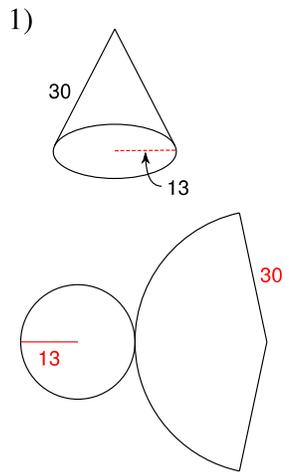


12)



Worksheet 48.3 Answers

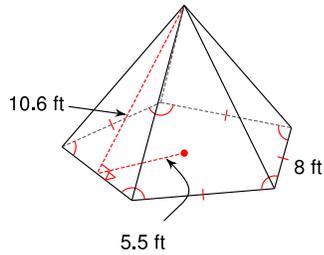
Copy the measurements given onto the net of each solid.



Lesson 48: Pyramids, Cones, and Spheres

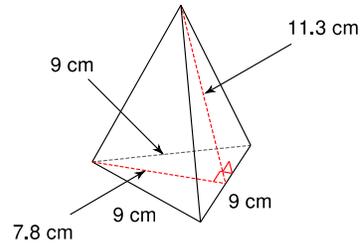
Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

7)



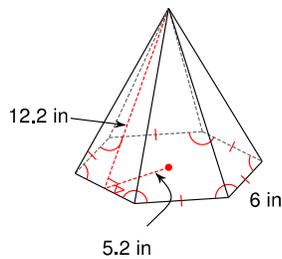
212 ft²; 322 ft²

8)



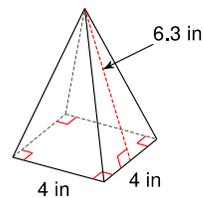
152.6 cm²; 187.7 cm²

9)



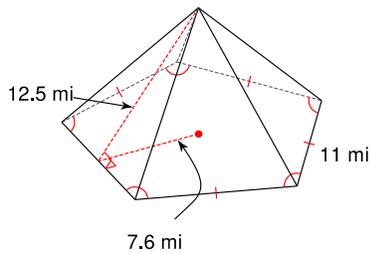
219.6 in²; 313.2 in²

10)



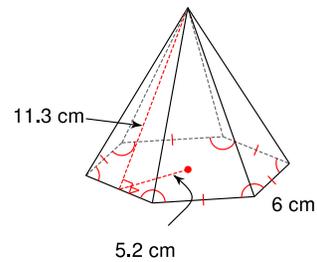
50.4 in²; 66.4 in²

11)



343.8 mi²; 552.8 mi²

12)



203.4 cm²; 297 cm²