LESSON 38: Evaluation of Functions

Lesson Summary: For the warm-up, students will solve a problem about study time. In Activity 1, they will see examples of graph transformations and graph a few equations. In Activities 2 and 3, they will do practice problems in the student book and the workbook. Activity 4 is an application activity related to the NFL draft. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 38:
- Video (length 23:00) on graphs of basic functions and their transformations. The video is required for teachers and optional for students.
- 1 Worksheet (38.1) with answers (attached)
- Application Activity
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Student Book (pages 80 – 81)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test* Workbook (pages 118 – 121)
- Note:
    - This lesson is challenging. The idea is for students to get a basic understanding of evaluating functions. They are not expected to master this topic in one lesson.

Objectives: Students will be able to:

- Answer the questions about a chart
- Graph parabolas
- Evaluate functions with relative minima and maxima and periodicity
- Analyze an application function about the NFL (National Football League) draft

ACES Skills Addressed: N, CT, LS
CCRS Mathematical Practices Addressed: Building Solution Pathways, Make Sense of Problems and Persevere in Solving Them
Levels of Knowing Math Addressed: Intuitive, Pictorial, Abstract, and Application

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 38 Warm-up: Solve the Study Time Questions	**Time: 5-10 Minutes**

Project the following table and sentences on the board:

X **Hours Student Studies for Test**	Y **Correct Number of Questions on Test**
2	18
3	27
4	36
5	45

A. Which of the following statements is false per the information in the table?

1. The student has to study at least 7 hours to get 63 questions correct on the test.
2. If he studies one hour, he will get 10 questions correct.
3. If he doesn't study, he will get all the questions incorrect.

B. What is the pattern you see in the table?
C. Write an equation to represent the information in the table.

Answers:
   A. *Number 2 is false.*
   B. *Pattern is 9 questions correct for every hour studied.*
   C. *Equation is 9x = y*

Lesson 38 Activity 1: Graphing Quadratic Equations and Their Transformations	**Time: 25 Minutes**

1. Students need to practice more basic graphs of functions before they can evaluate them. Graph each of the following on the board. Explain and have students give input as you draw each one.

2. Example A: Graph $f(x) = x^2 - 1$. Make a table first. Examples of input and output may be (-
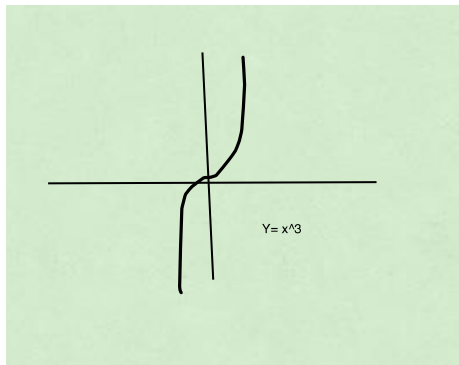
3,8), (-2,3), (0, -1), (2,3), (3,8). The graph should look similar to the one below. This shows that adding or subtracting a constant to the basic quadratic equations moves the parabola up or down on the y-axis.
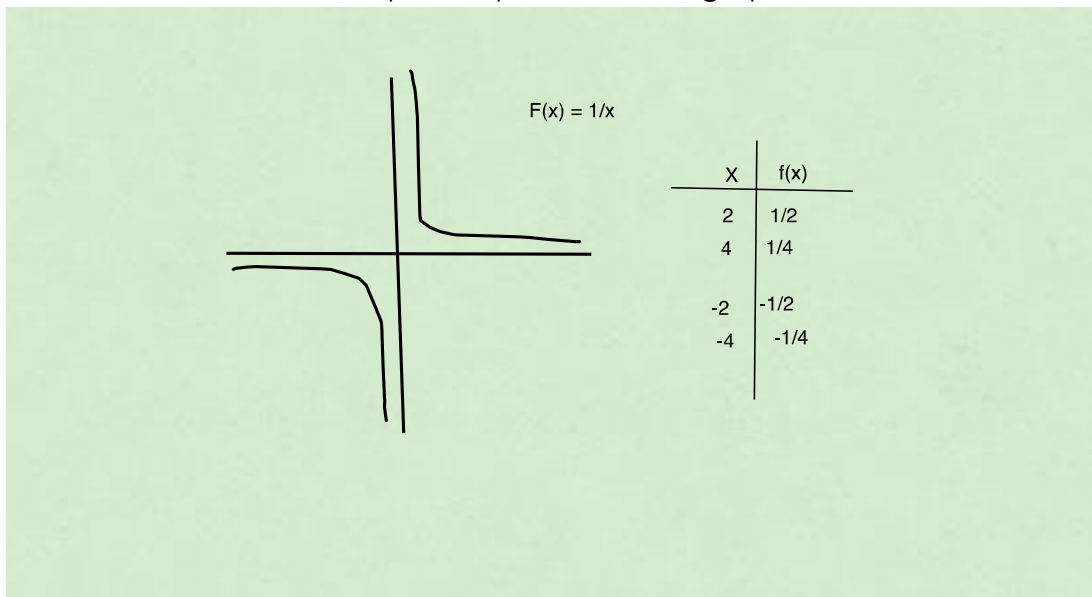
3.

Y= x^2

Y= x^2-1
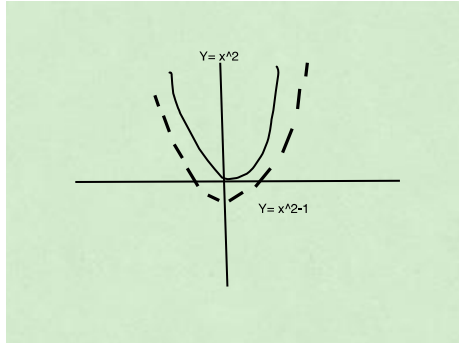
4. <u>Example B</u>: Graph $f(x) = x^3$. Some inputs and outputs may be (-2,-8), (-1,-1), (0,0), (1,1), (2,8). The graph should look similar to the one below:

5.

Y= x^3

6. <u>Example C</u>: Graph $f(x) = 1/x$. The inputs and outputs will show that f(x) approaches the x-axis but never touches it. The input/output table and graph should look similar to the one below:

F(x) = 1/x

X	f(x)
2	1/2
4	1/4
-2	-1/2
-4	-1/4

7.

8. Students can practice by graphing the four problems on **Worksheet 38.1**. Do the first one together if needed.
9. After the first problem is done, point out to students how the +1 in the equation $(x + 1)^2 - 2$ makes the graph move left one space and how the constant -2 makes it move down two on the y-axis.
10. <u>Note:</u> There are more practice problems available like these on math-aids.com.

Lesson 38 Activity 2: Evaluation of Functions	**Time: 20 Minutes**

1. Do the problems in the **student book pages 80 and 81**.
2. Look at the first graph on page 80. Explain the **relative minimum** and **relative maximum** in the graph. This means these points are the lowest and highest points for a portion of the graph. They are the "hills and valleys" in the graph.
3. Sometimes graphs have periodicity and they are called **periodic functions**. An example is the second graph on page 80, which has a period of 2. This means that it repeats at every 2 intervals on the x-axis.
4. Do questions 1 – 9 together.

Lesson 38 Activity 3: Independent Practice	**Time: 25 Minutes**

1. Do the problems in the **workbook pages 118 – 121.**
2. Explain the graph of where **functions are undefined** like the example graph on page 118.
3. The questions are challenging. Do the best you can in the time given.

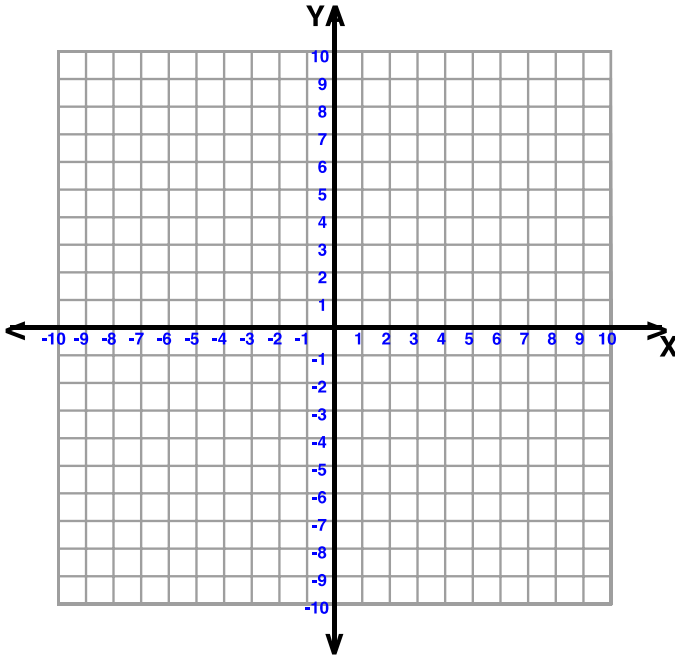| **Lesson 38 Application: NFL Draft Picks**	**Time: 25 Minutes**

1. Start with a discussion about the salaries of football players and what the draft is about. Some students may know a lot about this topic and they may be able to explain what the different rounds of the draft are and how they affect players' salaries.
2. Give the students a copy of the attached activity graph and questions. For better student copies, you may choose to print from this link.
3. Ask questions about what information is on the y-axis and the x-axis.
4. Discuss the questions in small groups.
5. There may not be time to finish and that is okay. The idea is to expose the students to a real-life application of the types of graphs they have seen in the book activities.
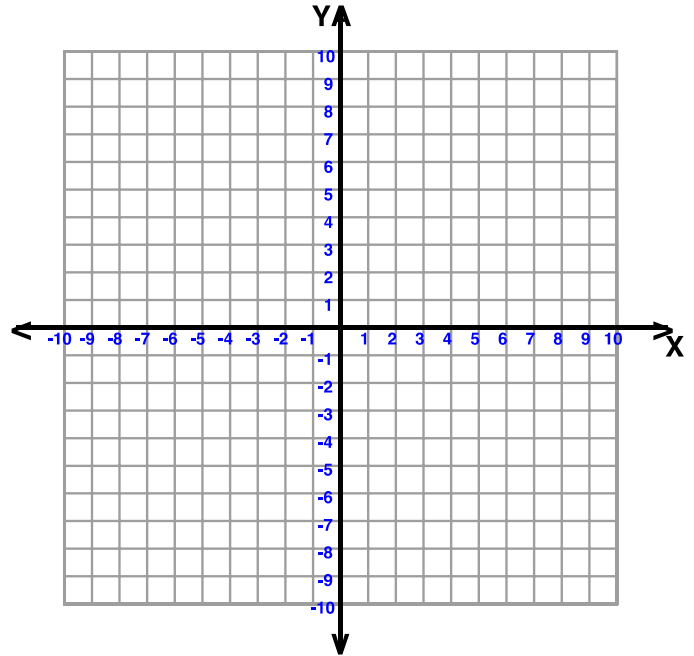
Lesson 38: Evaluation of Functions
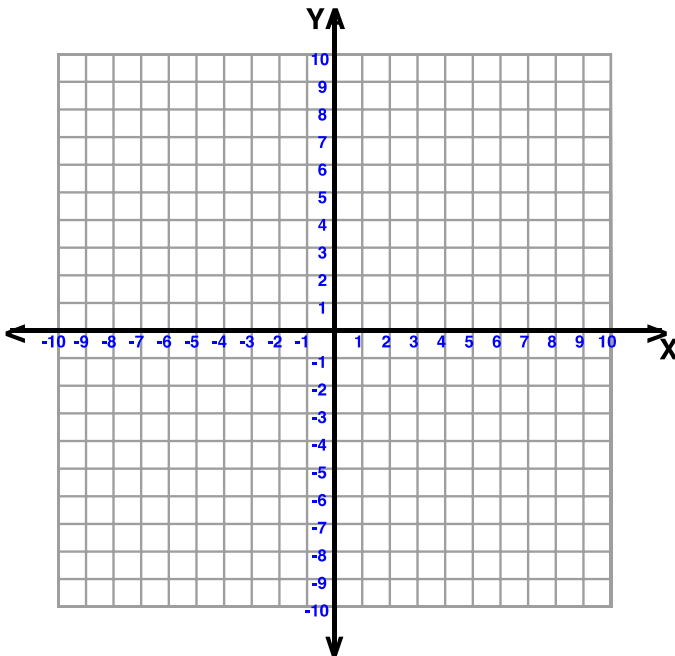
Worksheet 38A.1

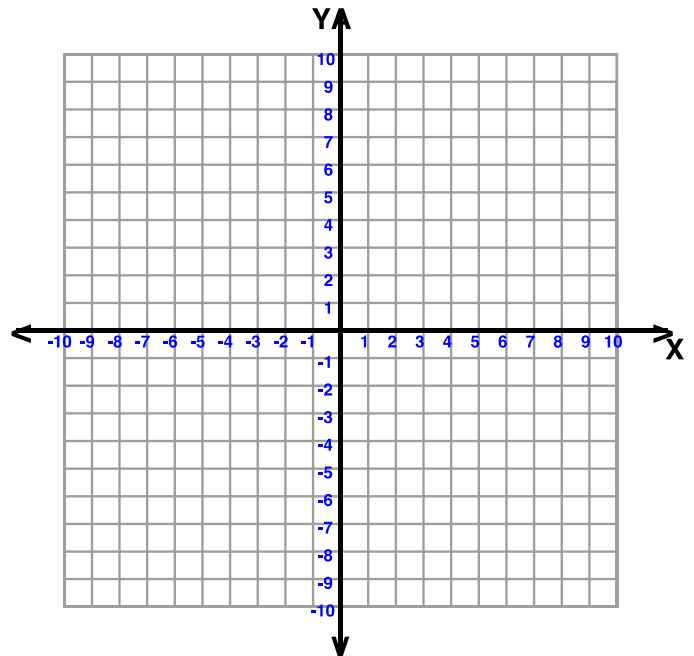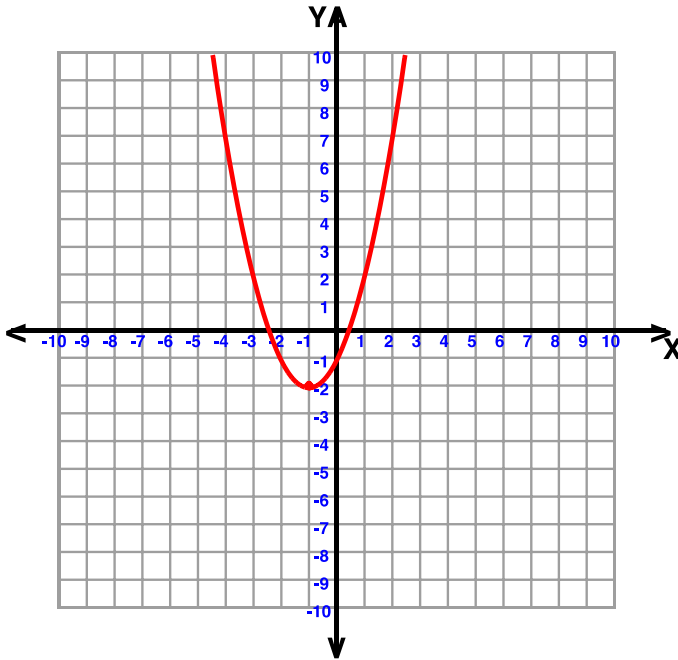   **Graph the given equation.**

1) $y = (x + 1)^2 - 2$

2) $y = -(x + 1)^2 + 2$
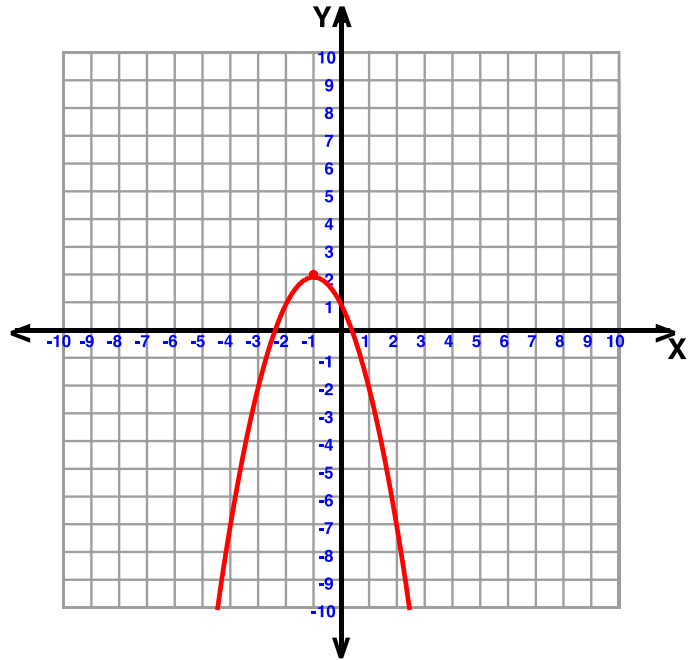
3) $y = (x + 2)^2 + 3$

4) $y = -2x^2 + 3$

Worksheet 38.1 Answers
Graph the given equation.
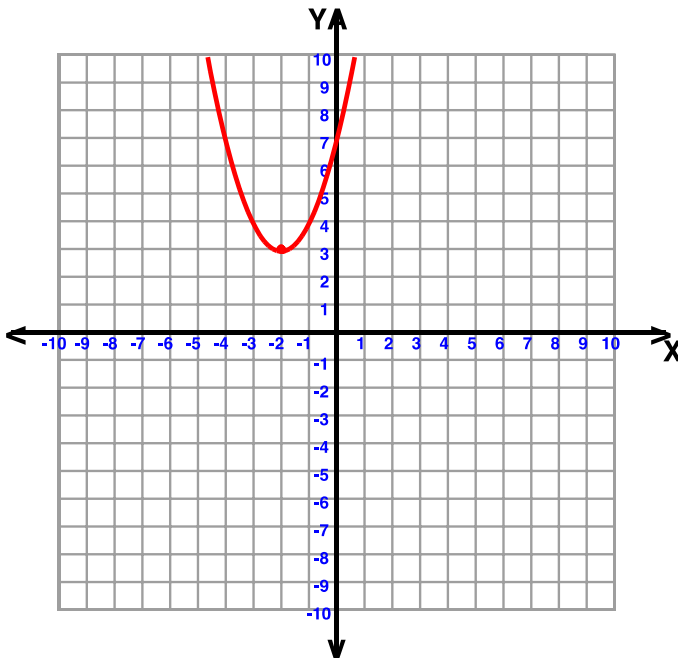
1) $y = (x + 1)^2 - 2$

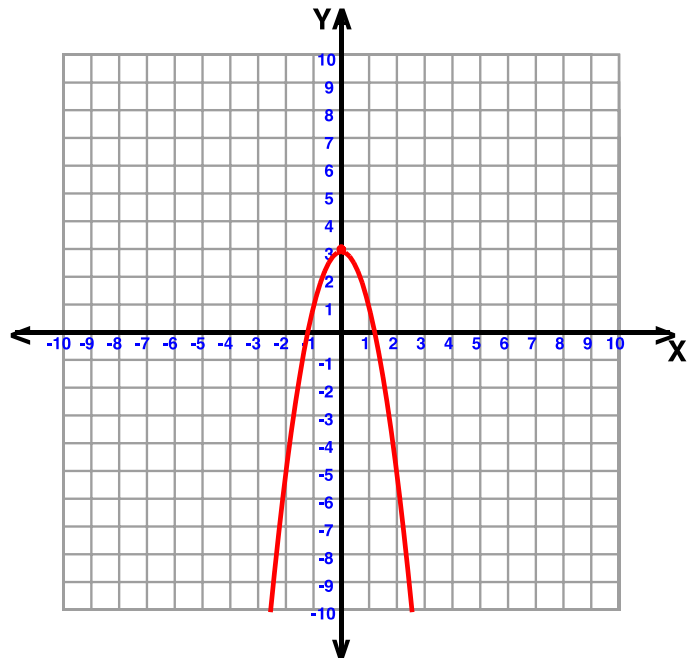2) $y = -(x + 1)^2 + 2$

3) $y = (x + 2)^2 + 3$

4) $y = -2x^2 + 3$

Application: NFL Draft

NFL Draft

Each year all 32 NFL teams replenish their team rosters with former college football players through the NFL draft. The NFL draft consists of seven rounds. Each team gets one pick per round, in reverse order of their success during the previous season. The least successful team picks first and the most successful team picks last. In addition to the 32 picks in each of the seven rounds, there are a number of compensatory picks given based off of free agency. These picks are awarded at the end of rounds 3 through 7.

College football players have a general idea of if and in what round they might be drafted, but even then there is often a range of a few rounds where a player might be drafted. Sometimes players don't end up being drafted near where they are projected at all. For example, in 2013, USC QB Matt Barkley was projected to be a first or early second round draft pic. He was not drafted until the 4th round. Should football players care where they are drafted? Shouldn't they simply be pleased to make an NFL team, do what they love doing and make a lot of money? Lets take a look at the four-year contracts that each draft pic signed after the 2013 draft. The graph below gives the four-year contract value by draft pic number.
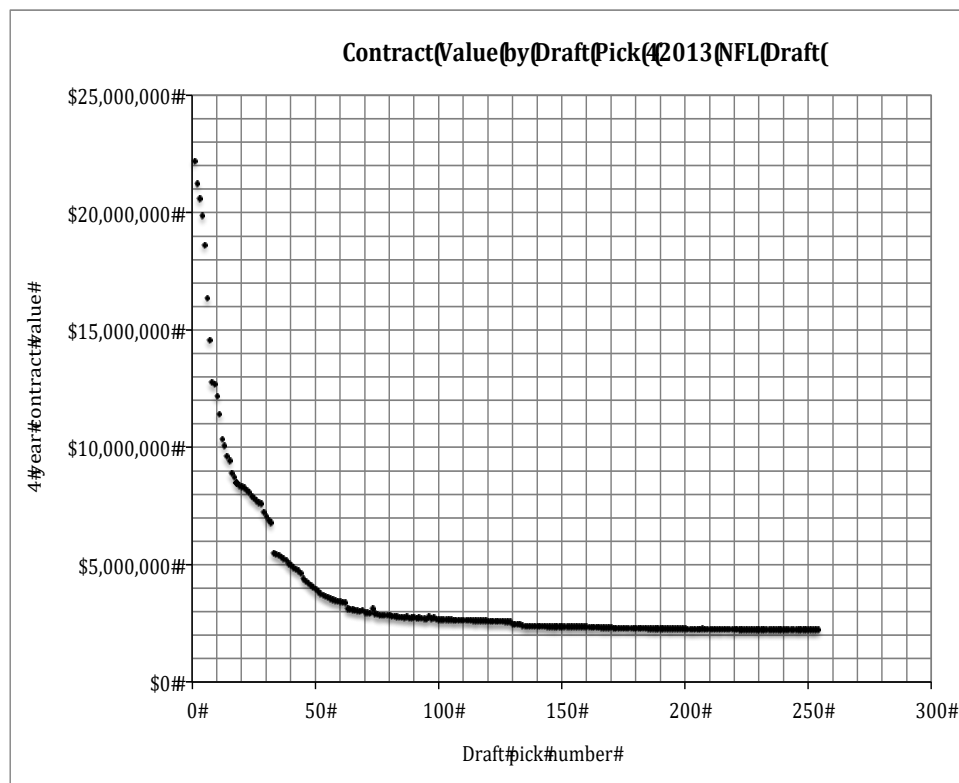
Contract Value by Draft Pick 2013 NFL Draft

1. Take a look at the graph. What do you see? What does it make you think? What does it make you wonder?

2. Describe the relationship between draft number and contract value. Be as detailed as possible.

3. I see many close clustered points, but then there is a gap or a jump in the data around x value 32 to 33. What might explain this gap or jump? What does that mean in terms of the context of the NFL draft?

4. As mentioned in the introduction, QB Matt Barkley was projected to be a first or early second round draft pic (there are 32 players selected in the first round and another 32 players selected in the second round). He slipped to the fourth round and was the 97th player selected. What did this slip in the draft cost him?

5. Players lose about 3% of their salary to agent fees, 25 to 35% to federal taxes, about 6% in state taxes, and Medicare at 1.5%. Pick a first round salary. How much is that player making on average per year over the 4 year contract? How much are they taking home per year after all taxes and fees?

6. Pick a player drafted near the middle or end of the draft. How much is that player making on average per year over the 4 year contract. How much are they taking home per year after all taxes and fees?

7. In general, how much would you say a player loses by sliding back a few draft picks? How much do they lose by sliding back a round or a few rounds? Should a player be concerned about where they are selected in the draft?

Brought to you by **Yummymath.com**

Application Answers

1. Take a look at the graph. What do you see? What does it make you think? What does it make you wonder?
   The later you are selected in the draft the less money you make. There is a steep drop off in pay during the first 30 or so picks. There is a greater difference in pay during the first couple rounds. There is less difference in pay in the later picks, say from pick 60 through 256. Why is the pay like this? Why is there such a drop off in pay during round one? Why is there a gap around pick 32?

2. Describe the relationship between draft number and contract value. Be as detailed as possible.
   The relationship is not linear. It might be modeled with exponential decay or a quadratic relationship. The later you are selected in the draft the less money you make.

3. I see many close clustered points, but then there is a gap or a jump in the data around x value 32 to 33. What might explain this gap or jump? What does that mean in terms of the context of the NFL draft?
   This is the difference between the last player picked in the first round and the first player picked in the second round. You can notice a huge drop off in pay.

4. As mentioned in the introduction, QB Matt Barkley was projected to be a first or early second round draft pic (there are 32 players selected in the first round and another 32 players selected in the second round). He slipped to the fourth round and was the 97th player selected. What did this slip in the draft cost him?
   The 9th player selected received a contract of $2,681,000. Depending on where he could have been selected in the first round he could have made anywhere from $6,700,000 to around $22,200,000. So he lost anywhere from $4,000,000 to $19,500,000. If he could have been picked in the early second round he would have made around $4,000,000 to $5,500,000. So he lost anywhere from $1,300,000 to $2,800,000. Students might also convert these answers to a per year amount.

5. Players lose about 3% of their salary to agent fees, 25 to 35% to federal taxes, about 6% in state taxes, and Medicare at 1.5%. Pick a first round salary. How much is that player making on average per year over the 4-year contract? How much are they taking home per year after all taxes and fees?
   Students' choices will vary. We've done one choice as an example.

   The 13th pick made about $10,000,000 over four years. That is 2,500,000 per year on average. Of that, he will lose about 46.5% to taxes and fees. 2,500,000 x 0.465 = $1,162,500. 2,500,000 - $1,162,500 = $1,337,500 per year. That's not too shabby but considerably less than the sound of a ten million dollar contract.

6. Pick a player drafted near the middle or end of the draft. How much is that player making on average per year over the 4-year contract. How much are they taking home per year after all taxes and fees?
   One example: Pick number 200 made about $2,250,000 over four years. This is $562,500 per year. Of that, he will lose about 46.5% to taxes and fees. 562,500 x 0.465 = $261,562.50. $562,500 - $261,562.50 = $300,937.50. It's a lot of money, but not nearly like the big contracts you hear about.

7. In general, how much would you say a player loses by sliding back a few draft picks? How much do they lose by sliding back a round or a few rounds? Should a player be concerned about where they are selected in the draft?
   Again student answers will vary considerably.
   Here's an example of what a student's work might look like.